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QUANTITATIVE METHODS OF ECONOMIC ANALISIS

TUTORIAL

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МИНИСТЕРСТВО ОБРАЗОВАНИЯ И НАУКИ РОССИЙСКОЙ ФЕДЕРАЦИИ

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КОЛИЧЕСТВЕННЫЕ МЕТОДЫ АНАЛИЗА ЭКОНОМИКИ

Учебно-методическое пособие
по дисциплине «КОЛИЧЕСТВЕННЫЕ МЕТОДЫ АНАЛИЗА ЭКОНОМИКИ»

Рекомендовано методической комиссией Института экономики и предпринимательства ННГУ для иностранных студентов, обучающихся по направлению подготовки 38.03.01 «Экономика» (бакалавриат) на английском языке

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В настоящем пособии изложены учебно-методические материалы по курсу «Количественные методы анализа экономики» для иностранных студентов, обучающихся в ННГУ по направлению подготовки 38.03.01 «Экономика» (бакалавриат).

Пособие дает возможность бакалаврам ознакомиться с современными методами количественного анализа экономических процессов, необходимыми для осуществления широкого спектра расчетов, с которыми сталкиваются экономисты-аналитики, бухгалтера и эксперты в банках, коммерческих организациях, страховых учреждениях при принятии решений об инвестировании, осуществлении той или иной коммерческой операции.

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CONTENT

INTRODUCTION

The theory and practice of financial and economic calculations have great importance in the preparation of economists, financiers, managers the study of

United influence of the financial and commercial operation at many processes makes the ultimate results unevident. For its estimation requires special quantitative analysis, whose main tasks are:

• identifying the dependence of the final results of the financial and commercial operations from its basic parameters, changing the relationship of these parameters, determining of their valid boundary values;

• finding equivalent changes in the conditions of these operations;

• measuring of the final financial results of operations for each of the participating parts;

• comparing the efficiency of various operations, etc..

This manual assumes a systematic presentation of the basic concepts and methods of financial calculations. It examines and identifies concepts such as interest rate, the discount rate, effective rate, operations, accumulation on simple and compound interest, methods of accumulating and discounting the payments, the principles underlying the credit calculations, etc.

The paper presents both basic and applied methods , algorithms of calculation of financial and commercial operations. The first group includes:

• operations accumulating and the discount using simple and compound interest;

• the calculation of key indicators of cash flows analyses in relation to irregular flows and different types of financial rent.

Among the applied methods of financial calculations presented:

• planning and evaluating the effectiveness of financial and commercial transactions;

• planning repayment of long-term debt;

• credit calculations;

• analysis and assessment of efficiency of investment projects, etc.

The peculiarity of all financial calculations discussed in the manual, is the time value of money, i.e. the principle of inequality of money relating to different points in time. It is assumed that the amount received today has a greater value than it received in the future.

Quantitative analysis of financial and commercial operations require the use of standardized models and methods of financial indicators calculation. Currently, the

financial calculations are usually performed on a computer using special software.

The manual will be useful to undergraduate and graduate students from all economic areas, studying the problems of the market economy and quantitative methods of analysis.

CHARPTER 1. FUNDAMENTALS OF FINANCIAL CALCULATIONS

1.1. THE TIME VALUE OF MONEY

In any financial calculations the amount of money is almost always associated with specific points in time or time interval. The time factor is no less important than the size of sums, therefore, contracts are always recorded the time, date, frequency of receipt of funds and their payment.

The well-known slogan "time is money" has a real basis upon which to determine the true value of money from the perspective of the current moment.

The importance of considering the time factor based on *the disparity of money relating to different points in time*: equal the amount of money (in the point of magnitude) "today" and "tomorrow" are evaluated differently. The marked dependence of the value of money from time to time is the results the following factors influence:

• money can make productively used as income-generating financial asset, i.e. the money can be invested and thereby "bring money";

• inflation processes lead the loss of purchasing power with time. Today on the ruble, you can buy more goods than tomorrow at the same ruble, because prices will rise;

• the uncertainty of the future and the associated risk increases the value of existing money.

There are two approaches and two corresponding types of economic thinking:

• static approach does not take into account the time factor. Using this approach allowed to operate with monetary indicators pertaining to different periods of time (carrying with them any arithmetic operations);

• the dynamic approach requires consideration of the time factor, so there is unlawful to carry out arithmetic operations with monetary values relating to different points in time.

At present, in both foreign and domestic practice prevails the second approach, taking into account the time factor when considering financial transactions. At the moment, it is developed convenient models and algorithms which allow to find out the true price of future revenues from the position of the current moment.

1.2. THE CONCEPT OF THE SIMPLEST FINANCIAL TRANSACTIONS

The simplest type of financial transaction (deal) - single lend of a certain amount of money (the debt) with the condition that after some time T will be

refunded the amount FV, $FV \geq PV$.

PV - present value of investments (amounts given in debt); FV - future value, accrued amount at the end of the financial transactions; T - duration of the financial transactions (in years, for example).

Capital gains creditor ($FV - PV$) is called interest money or percent.

The indicators characterizing the efficiency of the operation are:

• r_T - relative growth, interest rate;

• d_T - relative discount or discount (discount rate), expressing the rate of the value of money fall:

$$r_T = \frac{FV - PV}{PV}, (1.1)$$

$$d_T = \frac{FV - PV}{FV}. (1.2)$$

Values r_T and d_T characterize the capital growth of the creditor to the initial deposit (interest) or to the final amount (discount), i.e., represent the relative magnitude of income for a fixed period of time, which is called interest rates. They are measured in fractions of a unit or in percentage. Values r_T and d_T are interrelated in the sense that they can be expressed through each other.

The time interval, which is timed interest rate, called the interest period. As such period could be taken a year, half year, quarter, month or even day. In practice, often, deal with annual rates.

If T=1, then $r_T = r$, is called interest; $d_T = d$ and d is called the discount rate or the discount.

Interest rate is the amount of money that you must pay for the use of borrowed one monetary unit per period of time T=1.

To specify a simple financial transaction, you must know two of the following three values: PV, FV, r_T (d_T).

1.3. ACCUMULATION PROCEDURE FOR THE SIMPLEST FINANCIAL TRANSACTIONS

The process in which you set the original amount PV and r_T rate (or d_T) is the process of accumulation capital.

Economic sense of the operations: determine the magnitude of the sum, which the investor will or wishes to have at the end of the operation. If in the operation of the building is used interest rate (r), then the method is called recursive.

In this case, from (1.1) we get:

$$FV = PV \cdot (1 + r_T)$$

If the discount rate is used, the method is called anticipation. Then from (1.2) we get:

$$FV = \frac{PV}{1 - d_T}.$$

As r_T (or d_T) ≥ 0, i.e. $FV \geq PV$. We can figuratively conclude that the time generates money. The operation of the accumulation can be based on two schemes: the simple scheme (simple interest) and the scheme of compound interest (compound interest).

1.3.1. ACCUMULATION OF CAPITAL USING SIMPLE INTEREST RATE (RECURSIVE METHOD)

If invested capital increased annually by value $(PV \cdot r)$, i.e. the base interest does not change over time, such investment is made on terms of simple interest.

The following cases of interest capitalization exist:

a) $FV = PV \cdot (1 + r \cdot T)$ - a constant during the time T rate of interest .

b) $FV = PV \cdot (1 + \sum\limits_{i=1}^{T} r_i)$ - changes annual.

c) $FV = PV \cdot (1 + \sum\limits_{i=1}^{n} r_i \cdot t_i)$ - the interest rate r vary over different time intervals;

n - the number of rate changes during the fiscal operations, r_i - duration rates in years.

d) $FV = PV \cdot \left(1 + r \cdot \dfrac{T_{dn}}{K_0}\right)$ - the duration of operation is not an integer number of periods; K_0 is the number of days in a year, T_{dn} - the duration in days.

We should determine the duration of the operation adequate to the following rule: date of its beginning and end are considered to be 1 day. Thus there are three possible ways of calculating the duration of the operation:

1. English version: T_{dn} and K_0 are calculated exactly (using the calendar).

2. German version: apply conditional or fiscal year consisting of 360 days ($K_0 = 360$) and when calculating T_{dn} the month is considered as 30 days.

3. French version: dates of operation T_{dn} shall be calculated exactly with the calendar, and the duration of a year $K_0 = 360$ days.

Reinvestment under simple interest.

If at some point in time accrued amount (FV) is withdrawn and re-inserted together with interest at the simple interest, then the operation is called *reinvestment or interest capitalization*.

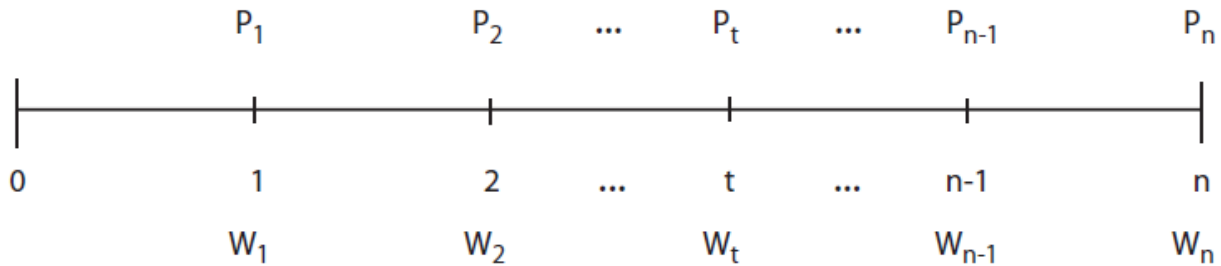It helps to draw out a time line and plot the payments and withdrawals accordingly.

Figure 1.1. Plot of payments.

Example 1.1.

An account was opend 20.01.2000 at a simple rate of 20% per year on the account, we invested amount of 5000 rubles. With 01.03.2000 rate of interest was changed and became 18%. 01.06.2000 was invested the amount of 10,000 rubles to the account. With 01.08.2000 rate of interest was changed and became 14%. 30.10.2000 the account was closed.

Find the sum thus obtained using the English and German.

Solve the problem taking into account the reinvestment 01.05.2000

Solution.

Draw the situation described on the time axis: note date changes in the terms of financial transactions and calculate the corresponding time intervals, using English simple interest.
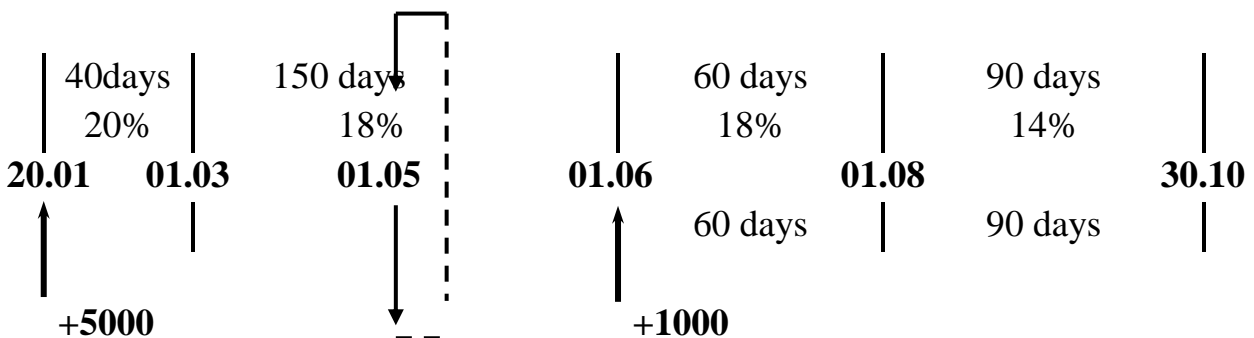

Figure 1.2. The graphical representation of the operation stages of the accumulation

of interest

Determine the amount received using English interest:

$$S(T) = 5000 \cdot \left(1 + 0,2\frac{40}{360} + 0,18\frac{150}{360} + 0,14\frac{90}{360}\right) + 10000 \cdot \left(1 + 0,18\frac{60}{360} + 0,14\frac{90}{360}\right) =$$

$$= 5661,11 + 10650 = 16311,11$$

Accrued interest in the amount of 5000 rubles for the entire period of operation and add the amount of 1000 rubles with accrued interest, starting from 01.06.

Determine the amount received using the German percents:

$$S(T) = 5000 \cdot \left(1 + 0,2\frac{40}{365} + 0,18\frac{153}{365} + 0,14\frac{91}{365}\right) + 10000 \cdot \left(1 + 0,18\frac{61}{365} + 0,14\frac{91}{365}\right) =$$

$$= 5661,37 + 10649,86 = 16311,23.$$

Determine the amount of money received, if the capital is reinvested 01.05.2000 using English interest. The time of reinvestment is shown at the time axis Fig. 1.2.

$$S(T) = 5000 \cdot \left(1 + 0,2\frac{40}{360} + 0,18\frac{60}{360}\right) \cdot \left(1 + 0,18\frac{90}{360} + 0,14\frac{90}{360}\right) + 10000 \cdot \left(1 + 0,18\frac{60}{360} + 0,14\frac{90}{360}\right) =$$

$$= 5261,11 \cdot 1,08 + 10650 = 16332.$$

1.3.2. ACCUMULATION OF CAPITAL USING COMPOUND INTEREST RATE

*(*RECURSIVE METHOD*)*

Investment is made in terms of *compound interest*, if the annual income is calculated not from the original value of the invested capital, but the total amount, including accrued and not demanded by the investor interest.

The following situations are possible:

a) Capitalization of interest once a year. In this case, the invested capital is equal to:

• in the end of the first year:

$$FV(1) = PV + PV \cdot r = PV(1+r);$$

• in the end of the second year:

$$FV(2) = FV(1) + FV(1) \cdot r = FV(1) \cdot (1+r) = PV(1+r)^2.$$

According to the method of mathematical induction we can prove that

$$FV(T) = PV(1+r)^T. \quad (1.3)$$

Therefore, the scheme of compound interest can be described by a geometric progression with the first member $a_1 = PV$ and the denominator $q = (1+r)$.

The expression (1.3) is called *the formula of compound interest accumulation* and a factor $(1+r)^T$ is called the coefficient of the accumulation (accumulation factor) of compound interest and denote A(r,T). This coefficient is equal to the accretion of one monetary unit at time T at a given interest rate r.

b) Accrual of interest m times during a year:

$$FV = PV \times \left(1 + \frac{r}{m}\right)^{mT}. \quad (1.4)$$

If the investment period T is not an integer, then, the first, the duration of operation should be determined in days, and then transformed into years as well as in the case of simple interest.

c) Quantity of periods (T) is not an integer number of periods. In this case, the following calculation method should be used:

dates of operation are $T = [T] + \{T\}$, where [T] is a whole years, and {T} is a part of the time period over a number of years.

Then $S(T) = S(0) \cdot (1+r)^{[T]} \cdot (1 + r \cdot \{T\})$.

d) Continuous charging.

In banking practice, especially using in electronic methods of financial transactions record, interest may be awarded for 1 day or even for a few hours. In this case we can talk about continuous compound interest.

Write the formula of the building at m accruals in the year:

$$FV = PV \times \left(1 + \frac{r}{m}\right)^{mT}.$$

Let , let's move this formula to the limit when :

$$\lim_{m \to \infty} PV\left(1 + \frac{r}{m}\right)^m = PV \lim\left(1 + \frac{r}{m}\right)^m = PV \cdot e^r, \quad (1.5)$$

$$\lim\left(1 + \frac{r}{m}\right)^m = e^r \text{ is the 2nd great limit, e} = 2,718281.$$

In order to distinguish a continuous rate from discrete rate r , it is denoted δ and is called the power of growth.

Then $FV(1) = PV \cdot e^{\delta}$, and $FV(T) = PV \cdot e^{\delta \cdot T}$. Discrete and continuous rates are functional dependencies between them because of the equality of the accumulation factors $(1+r)^T = e^{\delta T}$, of the formla follows:

$$\delta = \ln(1+r); \qquad r = e^{\delta} - 1.$$

1.3.3. THE ANALYSIS ACCUMULATION FACTOR

Theorem 1. The principle of market stability.

If you don't take into account taxes and other overhead costs, the accumulation factor at a certain interval is equal to the product of the accumulation factors on each of its constituent potential:

$$A(0,k) = \prod_{n=1}^{k} A(n-1,n) \,.. \,(1.6)$$

The proof is based on the method of mathematical induction.

It is important to note that in this simple formulation of this theorem gives only approximate results in financial practice, as it doesn't take into account many factors: taxes, fees, renewal of contracts, etc.

The magnitude of the accumulation factors of the procedure depends on the interest rate r and the length of the interval.

Example 1.2.

Calculate the accumulation factor at a rate of r=0.2 and time base of 360 days. Compare the coefficients of building on simple and compound interest.

	30 days	60 days	90 days	180days	1 year	2 years	5 years	10 years
$1+rT$	1,017	1,033	1,050	1,100	1,200	1,400	2,000	3,000
$(1+r)^T$	1,015	1,031	1,047	1,095	1,200	1,440	2,488	6,192

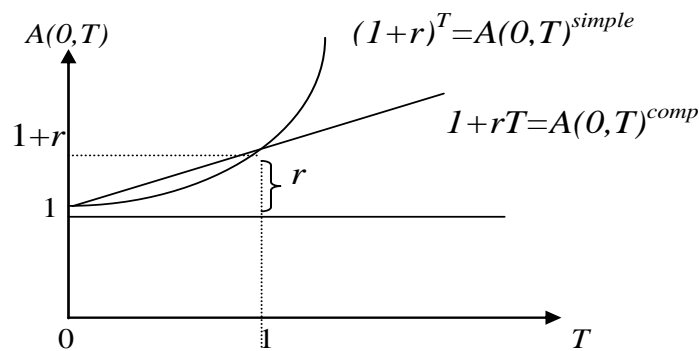Give a graphic illustration of the changes of the accumulation factors.

Fig.1.3. The coefficients change building on simple and compound interest

It is obvious that when T=1 the coefficients are the same and equal to 1+r. It can be shown that for any r, depending on the values of T, we have the following inequality:

$$1 + r \cdot T > (1+r)^T \quad , \quad 0 < T < 1;$$

$$1 + r \cdot T < (1+r)^T \quad , \quad T > 1.$$

Therefore, when the accretion complex percent more profitable for the investor.

1.3.4. ACCRETION OF CAPITAL BASED ON THE ANTICIPATING METHOD

If anticipating method it is also possible to use two schemes building: the scheme is simple and compound interest.

May d - annual discount rate. From formula (1.2), we obtain:
$PV = FV \cdot (1 - d_T)$, then

$$FV = \frac{PV}{1 - d_T}. \ (1.7)$$

Depending on schema building d_T can take different values:

a) Accrual of interest 1 time per year on a simple interest:

.

In this case, must satisfy the condition: , ie .

b) Accrual of interest for the period less than one year (or if not the whole, T):

$$FV = \frac{PV}{1 - T \cdot d}.$$

c) Calculation of compound percent 1 time per year:

$$FV = \frac{PV}{(1 - d)^T}.$$

d) Calculating a compound percent m times per year:

$$FV(T) = \frac{PV}{(1 - \frac{d}{m})^{mT}}.$$

1.3. THE CONCEPT OF AN EQUIVALENT INTEREST RATES. EFFECTIVE RATE.

Interest rates are called equivalent, if their application gives the same end result of the accumulation (the same sum FV) from the same initial sum PV. To find the equivalent rate we should equate accumulation factor in two schemes: using equivalent rate and initial.

Accumulation factor is called value, which is multiplied by the initial value PV. Let us denote it $A(r,T)$.

Example 1.3.

To find the monthly rate of interest equivalent to an annual rate of compound interest rate r.

The accumulation factor in the schema of compound interest: $A(r,T) = (1+r)^T$
The factor taking into account calculating interest 1 time per month $A(r,T) = (1+r_{mes})^{12 \cdot T}$. Equate multipliers building and find that $r_{mns} = \sqrt[12]{r} - 1$.

Effective rate.

Definition: the annual rate of compound interest giving the same ratio between the sum of PV and the sum of FV in any payment scheme is called the effective rate and is denoted by r_e.

By definition effective rate $FV = PV \cdot (1 + r_e)^T$. Here

$$r_e = \left(\frac{FV}{PV}\right)^{\frac{1}{T}} - 1. \ (1.8)$$

You can find your effective rate of interest for any scheme, using the definition of equivalence in interest rates. For example, for a scheme of compound interest accrued m times in a year:

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1. \ (1.9)$$

Example 1.4.

Compare the performance of the following schemes accumulation capital:

a) $r = 20\%$; $m = 4$;

b) $r = 22\%$; $m = 2$.

Calculate the effective rate for both cases:

a) $r_e = (1 + 0{,}2/4)^4 - 1 = 0{,}2155$

b) $r_e = (1 + 0{,}22/2)^2 - 1 = 0{,}2321$

Therefore, the scheme (b) will be more profitable for investors.

1.4. DISCOUNTING

Discounting task is reverse to the accumulation of interest: using known sum *FV* that should be paid after time T, it is necessary to calculate the sum *PV* (the loan).

The term "discounting" is used in a broader sense too - a mean of determining any value related to the future at some earlier point in time. This method is often referred to as conversion value to some (usually initial) time.

The value *PV* found using the discounting is called the present value of future sum *FV*, or modern (current, capitalized) value.

Depending on the interest rate it is possible to use two methods of discounting:

mathematical discounting and banking (commercial) accounting. Interest rate r is used in the first case, the discount rate d – in second.

1.4.1. MATHEMATICAL DISCOUNTING

In this case, the task can be formulated as follows: what is the initial sum *PV* you can give as credit that allows to take the sum *FV* in the end on the term of accumulation of interest.

Interest calculation can be done according to the scheme simple and compound interest.

a) accumulation of simple interest:

$$FV = PV \cdot (1 + r \cdot T) \rightarrow PV = \frac{FV}{1 + r \cdot T},$$

T is measured in years

b) accumulation of compound interest capitalizing once a year:

$$FV = PV \cdot (1 + r)^T \rightarrow PV = \frac{FV}{(1 + r)^T}.$$

c) Accrual in the scheme of compound interest once a year:

$$FV = PV \cdot (1 + r/m)^{m \cdot T} \rightarrow PV = \frac{FV}{(1 + r/m)^{m \cdot T}}.$$

Accumulation factor by which is multiply by the final sum FV is called the discount multiplier (coefficient). Let us denote it $s(r, T)$. For example, in case (a):

$$s(r, T) = \frac{1}{1 + rT}, \text{in case (b):} \ s(r, T) = \frac{1}{(1 + r)^T}.$$

1.4.2. BANKING ACCOUNTING. CONSIDERING OF BILLS

The bill (the promissory note) is drafted in statutory form unconditional written debt monetary obligation (securities), confirming the unconditional obligation of the Issuer to pay at maturity a certain amount to the payee or the person indicated by him.

Bills can be simple and transferable.

A promissory note is a written document that contains simple and unconditional obligation of the Issuer (the debtor) to pay a certain sum of money at a certain time and place to the payee or to his order.

Two parties involves in this bill: the Issuer, which itself is directly and unconditionally undertakes to pay on issued promissory note, and the note holder, who owns the right to receive payment on the bill.

A bill of exchange (draft) is a written document containing an unconditional

order of the Issuer to the payer (drawee) to pay a certain sum of money at a certain time and place to the payee or to his order.

Discounting bills of exchange consist of such possess as: the payee transfers (sells) a promissory note to a Bank or any other financial institution before the date of payment and receives a sum less percent (for early receipt).

Thus, the owner of a bill through its sale has the ability to get less sum of money but ahead of schedule.

When accounting for bills of exchange banking or commercial account is applicable. According to this method, the interest for using the loan (in the form of a discount) will accrue on the payable sum at maturity (*FV*). When this is applied, the discount rate is called d.

The discount rate depends on the quality of notes, the period prior to its maturity . The discount rate is determined in the contract between the payee and the Bank. The most often used is the simple interest rate, but it is possible to use compound interest too.

The promissory note may be with a fixed amount of the debt or accumulated interest. The amount of discount is determined by the formula (for promissory notes, which specify a fixed amount of debt, without interest):

$$D = FV \cdot d \cdot \frac{T_{days}}{K_0},$$

D – the amount of the discount; FV – promissory note amount; T_{days} - the number of days to maturity; d – the discount rate of the Bank; K_0 - time base in days.

Received by payee amount: $PV = FV(1 - d \cdot \frac{T_{dn}}{K_0})$.

When discounting compound percent discount factor: $(1-d)^T$.

Banks usually accept the promissory note, with obligations respectable and well-known firms, which solvency is not in doubt, the so-called first-class bills. Promissory note obligations small and financially weak firms are considered usually highly inflated interest rates.

Example 1.5.

What amount you borrowed, if you have the opportunity in three years to pay the sum of 150,000 rubles, the Bank offers interest rate of 6% per annum with quarterly compounding.

To determine the amount of debt will transact mathematical discounting:

$$PV = \frac{150000}{(1 + 0,06/4)^{4 \cdot 3}} = 125458, bub.$$

Example 1.6.

20.12. 2012 presented for consideration the bill in 2 million rubles, maturity bills 30.03.2013. The Bank purchase the bill at the rate of 18% simple percent (English version). What amount will be given to the owner of the promissory note.

Calculate how many days before maturity presented the bill according to the calendar of - 100 days.

$$PV = FV \cdot (1 - d \cdot \frac{T_d}{K_0}) = 2 \cdot (1 - 0,18 \cdot \frac{100}{365}) = 1,9014$$

1.5. FINANCIAL EQUIVALENCE OBLIGATIONS.

Often, there are cases when you need to replace one monetary obligation to the other in practice of financial calculations. For example, payment should have longer period of payment, we need to combine multiple payments into one, and so on. Such problems are solved on the basis of the principle of fiscal equivalence of obligations.

Equivalent are the payments that are equal at a single point in time. Casting is done by discounting (cast to an earlier date) or, on the contrary, the procedure of accumulation (if the date is in the future).

Example 1.7.

There are two obligations:

1. to pay 500 thousand rubles in 6 months;

2. to pay 400 thousand rubles in 8 months.

Can we consider them as equivalent?

The payments provide a simple rate of interest r = 15% per annum.

Find modern value of payments on the basis of discounting.

$$PV_1 = \frac{500}{(1 + 0,15 \cdot 6/12)} = 465,12 \quad PV_2 = \frac{600}{(1 + 0,15 \cdot 8/12)} = 545,45$$

Modern cost payments are not equal. Therefore, they are not equivalent.

1.5.1. CONSOLIDATION (MERGING OF PAYMENTS)

A common method of solving such problems is the development of the equation of equivalence, in which the sum of replaceable payments (given to any point in time) is equal to the sum of the payments under the new commitment (shown by the same date). For short-term obligations, the conversion is usually done on the basis of simple interest, and for others are complex interest rates. Typically, for a

combined payment set all parameters, except one, in defined, and the object is to determine the unknown parameter.

Example 1.8.

Two payment 2 million and 3 million rubles with the terms of payment of respectively 200 and 320 days are combined into one payment with a period of 250 days. The parties agreed to use simple conversion rate 10 %.

Find a consolidated amount of debt.

Let's choose 250 days as the base date. Let's calculates payments to the selected date. For the first payment must be increased, and for the second - discounting.

$$S_1 = 2 \cdot \left(1 + 0,1 \cdot \frac{50}{360}\right) = 2027778 \, \text{RUB} \quad S_2 = \frac{3}{\left(1 + 0,1 \cdot \frac{70}{360}\right)} = 2942779 \, \text{RUB}$$

Then a consolidated payment will be equal to their sum:

$$S = S_1 + S_2 = 4970557 \, \text{RUB}$$

Example 1.9.

Sums of 20, 40 and 15 million rubles must be paid in 100, 180 and 150 days, respectively. The parties agreed to replace them with a single payment of 70 million rubles at the rate of 10% simple percent.

Find payment period.

Find modern value of all payments and make an equivalence equation.

$$\frac{20}{\left(1 + 0,1 \cdot \frac{100}{365}\right)} + \frac{40}{\left(1 + 0,1 \cdot \frac{180}{365}\right)} + \frac{15}{\left(1 + 0,1 \cdot \frac{150}{365}\right)} = \frac{70}{(1 + 0,1 \cdot T)}$$

19,467+38,12+14,408=71,995 million rubles is the sum of the blended payment.

$$71,995 = \frac{70}{(1 + 0,1 \cdot T)} \rightarrow T \approx 1,11 \, \text{T year or 405 days.}$$

1.5.2. CONVERSION AND ACCUMULATION OF CAPITAL

We are talking about combining the conversion (exchange) exchange and accumulation of interest. There are two sources of income: change rate and the accumulation of interest in the operation of the building with the conversion of currencies. Moreover, if the second of these unconditional (the interest rate is fixed), then this couldn't be said about the first. Moreover, the double conversion exchange could be unprofitable. Different versions of conversion are shown at figure 1.3.
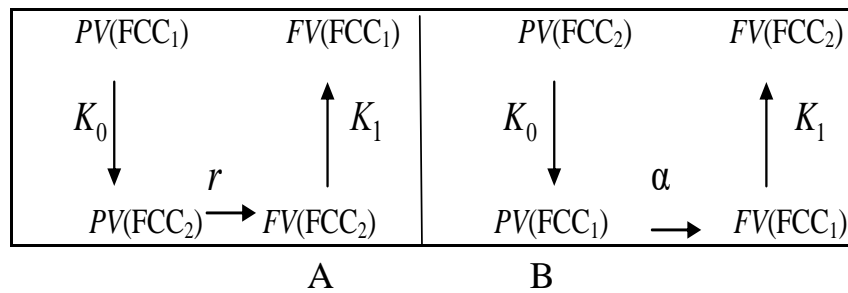
Fig. 1.4. The scheme of accumulation of capital with conversion

Consider (A): the procedure of capitalization with conversion using scheme of simple interest. We adopt the following notation:

- $PV(FCC_1)$- the present value of deposit in freely convertible currency;
- $PV(FCC_2)$- the present value of deposit in national currency;
- $FV(FCC_1)$- the future value of deposit in freely convertible currency;
- $FV(FCC_2)$- the future value of deposit in national currency;
- K_0 - the exchange rate at the beginning of the operation (the course of FCC_1 in national currency);
- K_1 - the exchange rate at the end of operation;
- T - term of the deposit;
- r – interest rate for national currency deposit;
- α – interest rate for freely convertible currency.

The operation involves three steps:

1. currency exchange for national currency;
2. capitalization of the sum;
3. the conversion of the original currency.

The final (future) value, measured in foreign currency, is defined as

$$FV(\text{CKB}) = PV(ACC_1) \cdot K_0 \cdot (1 + T \cdot r) \cdot \frac{1}{K_1}. \quad (1.10)$$

Three factor of this formula correspond to the above-mentioned three steps of the operation. The accumulation factor takes into account double conversion:

$$A(r,T) = \frac{K_0}{K_1} \cdot (1 + T \cdot r) = \frac{1 + T \cdot r}{K_1/K_0} \qquad = (1.11)$$

The interaction of the two factors of growing of PV is presented the most clearly. With a growth rate the factor increases linearly, in turn, the growth of final course reduces its:

the derivative of the multiplier in this course: $\dfrac{dA}{dK_1} = -\dfrac{K_0}{K_1^2}(1 + Tr) < 0$.

Thus, to determine if the operation with conversion is more profitable, it is

necessary to compare it with the usual accumulation factor in original currency and to determine when the final sum will be bigger. This is done by comparing the accumulation factors of these operations.

If the operation with conversion is the best, i.e., $\dfrac{K_0}{K_1} \cdot (1 + Tr) > (1 + T \cdot \alpha)$,

$k = \dfrac{K_1}{K_0}$ is a quantity that characterizes the ratio of exchange rates. Because at the moment of conclusion of the contract K_1 is unknown, it is useful to determine the maximum allowable its value at which level the efficiency will be equal to the prevailing rate for deposits in hard currency and the use of double conversion provides no additional benefits. This would equate multipliers building:

$$(1 + T \cdot \alpha) = \frac{K_0}{K_1}(1 + T \cdot r). \quad (1.12)$$

Here we find:

$$K_1 = K_0 \cdot \frac{1 + T \cdot r}{1 + T \cdot \alpha}. \ (1.12)$$

Thus, if the operation with conversion is more profitable: $K_1 < K_0 \cdot \dfrac{1 + Tr}{1 + T\alpha}$.

When utilizing compound interest the formula of the accumulation should be changed:

$$FV(\text{СКВ}) = PV(CKB) \cdot K_0 \cdot (1 + r)^T \cdot \frac{1}{K_1}. \ (1.13)$$

The accumulation factor can be written (with regard to dual conversion):

$$A(r, T) = \frac{K_0}{K_1} \cdot (1 + r)^T = \frac{(1 + r)^T}{k} \quad \text{(k is the growth rate of the exchange rate)}.$$

Determine the profitability of the operation as a whole for the owner of the exchange on the basis of the effective rate of interest r_e. Turning to the effective rate formula (1.7) and substituting it in (1.13) we get:

$$r_e = \frac{1 + r}{\sqrt[T]{k}} - 1. \ (1.14)$$

This formula shows that the efficiency of operation is determined by the ratio of the annual accumulation factor using the adopted interest to the average annual change rate, i.e., with increasing growth of exchange rate the effectiveness of conversional operations falls.

Example 1.10.

Will it benefit the scheme of capitalization with conversion of rubles into dollars under the following conditions: $K_0 = 30$ руб; $r = 10\%$ with monthly compounding. Deposit rate in dollars $\alpha = 3,5\%$ per annum with monthly compounding. The transaction period of one year. Projected exchange rate up to 30.5 rubles per dollar.

Compare the multipliers of the building on both schemes:

$$A(0,1)^{kon} = \frac{1}{K_0} \cdot \left(1 + \frac{\alpha}{12}\right)^{12} \cdot K_1 = \frac{1}{30} \cdot \left(1 + \frac{0,035}{12}\right)^{12} \cdot 30,5 = 1,0528.$$

$$A(0,1) = \left(1 + \frac{r}{12}\right)^{12} = \left(1 + \frac{0,1}{12}\right)^{12} = 1.1047.$$

Accumulation factor is bigger when the accretion of capital in the ruble Deposit. Therefore, under these conditions, the conversion is not profitable. Find at what exchange rate operation of conversion may be advantageous to the investor. To do this, determine: what K_1 is the multiplier of the building in operation with the conversion would give more appropriate multiplier than in operation without conversion.

$$\frac{K_1}{30} \cdot \left(1 + \frac{0,035}{12}\right)^{12} > \left(1 + \frac{0,1}{12}\right)^{12}, \quad K_1 > \frac{1,1047}{1,0356} \cdot 30 = 32,00, \text{ RUB}$$

Thus, the conversion will be beneficial only if the exchange rate over the year will be more than 32 rubles.

CHAPTER 2. INFLATION.

2.1. BASIC CONCEPTS

Inflation is characterized by a depreciation of the national currency (i.e. reducing its purchasing power) and a general increase of prices in the country.

The inflation measure based on the inflation rates, which characterize the change in the price level for a fixed set (basket) of goods and services for a certain period of time. For example, the inflation index (the growth index of consumer prices) per week, month, or year, calculated on the basis of the consumer basket for different regions of the country. In addition it is calculated the deflator for the various branches of production and gross national product (GNP) of the country for various periods of time (month, quarter, half year, year).

The inflation rate depends on the structure of the "basket" of goods on the basis of which the changes in prices is analyzed in the economy. Distinguish:

• the consumer price index in the basket includes only consumer goods;

• the index of prices of production: calculations are made for specific groups of goods (raw materials, agricultural products, transportation charges etc).

• deflator index in the basket includes all product groups.

The following types of inflation: current; cumulative and average for the period (annual, monthly, etc).

The level of current inflation shows the percentage of changed rates compared with the previous period.

Consider a basket of k commodities, each of which is included in the basket in the number of units q_i and the price for the moment t is $p_i(t)$, $i = \overline{1,k}$ currency units per unit of product. Then the value of the basket at the moment t is

$$X(t) = \sum_{i=1}^{k} p_i(t) \cdot q_i. \tag{2.1}$$

Definition: the inflation rate (consumer prices) during the time from t_1 to t_2 is called the dimensionless quantity:

$$J(t_1, t_2) = \frac{X(t_1)}{X(t_2)}, \quad t_2 > t_1, \tag{2.2}$$

and the inflation rate for this period is called the value:

$$H(t_1, t_2) = \frac{X(t_2) - X(t_1)}{X(t_1)} = J(t_1, t_2) - 1. \tag{2.3}$$

From the definition it follows that

$$J(t_1, t_2) = 1 + H(t_1, t_2). \tag{2.4}$$

The inflation index *I* shows how much prices have increased over the period, and the rate of inflation H (after multiplying by 100) is the percentage increased of prices.

Select the base unit of time natural for the purposes of the studies, and denote I_t the index and $H(t)$ - the rate of inflation over the unit interval:

$$J(t) = \frac{X(t)}{X(t-1)}, \quad and \quad h(t) = J(t) - 1. \ (2.5)$$

It follows:

$$X(t) = X(t-1) \cdot J(t) = X(t-1) \cdot [1 + H(t)], \ (2.6),$$

i.e. increasing of prices per one period $(t-1, t)$ occurs according to the scheme of compound interest, where the interest rate does the rate of inflation . We can say that the inflation rate characterizes the rate of growth of prices for the chosen unit of time.

Theorem. If $t_0 < t_1 < t_2 < ... < t_n$, the inflation index in the interval (t_0, t_n) is equal to the product of the inflation indexes on each of its constituent subintervals (i.e. the index of prices for several periods is equal to the product of chain price indexes)

$$J(t_1, t_n) = 1 + H(t_1, t_n) = \prod_{i=1}^{n}(1 + H(t_{i-1}, t_i)). \qquad (2.7)$$

2.2. INTEREST RATES AND INFLATION

Barrier rate.

Consider the process of building capital in conditions of inflation.

Without inflation-adjusted initial sum *PV* at a given rate of interest becomes *FV* in a certain period *T* and represents a real value of capital. In terms of inflation the real value of *FV* will be less due to recent price increases. We denote the real value of accumulated capital in the conditions of inflation FV_H. Then $FV_H = \frac{FV}{J(T)}$. Let accumulation of capital carried out on simple interest, then

$$FV = PV \cdot (1 + r \cdot T), \text{ and } FV_H = \frac{FV}{J(T)} = \frac{PV \cdot (1 + rT)}{J(T)}. \ (2.8)$$

To keep the initial cost of capital in conditions of inflation requires to have the accumulation factor in (2.8) equal to one.

$$\frac{(1 + rT)}{J(T)} = 1. \ (2.9)$$

We denote the rate of interest that provides this ratio r^*, and call barrier rate.

Thus, barier rate is the annual interest rate that inflation allows you to save the initial cost of capital. From (2.9) we can find:

$$r^* = \frac{J(T)-1}{T}. \ (2.10)$$

When calculating a complex percentage multiplier of the building, ensuring the preservation of the original cost of capital: $\frac{(1+r)^T}{J(T)} = 1.$

Then the barrier rate will be determined by the formula:

$$r^* = \sqrt[T]{J(T)} - 1. \ (2.11)$$

Can be similarly obtained barrier rates and any other schemes of building capital.

All interest rates are called negative interest rates if they are lower than barrier. Their application leads to the erosion of capital. Rates that are above barrier, they provide some capital growth in the conditions of inflation and are called positive interest rates.

Example 2.1.

Let $r = 10\%$, H= 12%. Find annual bet barrier:

$$r^* = \frac{J(T)-1}{T} = \frac{1{,}12-1}{1} = 0{,}12.$$

Therefore, the interest rate $r = 10\%$ is negative interest rate and the accretion of capital at such rate can be considered only as an option to reduce losses in inflation.

Gross rate (interest rate, taking into account inflation).

In terms of inflation considering the interest rate which provides the desired return on capital. Such rate is called gross rate or rate taking into account inflation. Denote it by r_H.

As already mentioned, the real value of accumulated capital will be less than the nominal value FV in the conditions of inflation. To restore the real value of accrued capital, i.e., to yield the operations building was equal to r, it is necessary to increase the sum FV in $J(T)$ times, where $J(T)$ is the inflation rate for this period. Then

$$\widetilde{FV} = PV \cdot J(T) = PV \cdot A(r,T) \cdot J(T). \ (2.12)$$

$\widetilde{FV}$ is the accumulated sum which provides the desired yield, $A(r,T)$ is the accumulation factor with lending rate r for the period T.

Adequate tithe rule of fiscal equivalence:

$$A(r,T) \cdot J(T) = A_H(r,T) \,,, \quad (2.13)$$

$A_H(r,T)$ is the multiplier of the building, using the gross rate r_H.

Considering the different schemes of interest, from (2.13) we get the formula for calculating the gross rates for these schemes. You should always use the inflation index for the whole period.

For a simple interest rate equation (2.13) can be written:

$$(1 + T \cdot r) \cdot J(T) = (1 + T \cdot r_H),$$

from which we obtain:

$$r_H = \frac{(1 + T \cdot r) \cdot J(T) - 1}{T}. \quad (2.14)$$

For the case of compound interest are: $(1+r)^T \cdot J(T) = (1+r_H)^T$, hence

$$r_H = (1+r) \cdot \sqrt[T]{(J(T))} - 1. \quad (2.15)$$

If the accrual of interest occurs once a year, use the appropriate formula of acumulation:

$$\left(\left(1 + {}^{r_H}\!/\!_m \right)^{mT} \right) = \left(1 + {}^{r}\!/\!_m \right)^{mT} \cdot J(T),$$

hence we obtain:

$$r_H = m \cdot \left(\left(1 + {}^{r}\!/\!_m \right) \cdot {}^{mT}\!\sqrt{J(T)} - 1 \right). \quad (2.16)$$

You can get similar formulas when you use an discount rate d.

2.3. THE FISHER FORMULA

Let's select the base unit of time is one year ($T = 1$) and find the relationship between the interest rate (real rate of return) and the gross interest rate (nominal rate). For this, we will substitute $T = 1$ in any of the formulas (2.14) or (2.15) (with accumulation factoron simple and compound interest is the same).

$$r_H = \frac{(1 + T \cdot r)J(1) - 1}{T} = \frac{(1 + r)(1 + H(1)) - 1}{1} = (1 + r) \cdot (1 + H) - 1 = r + H + rH.$$

Therefore,

$$r_H = r + H + rH. \quad (2.17)$$

The relation (2.17) is called the formula I. Fisher, widely used in the financial analysis for the conversion of the nominal and real rates.

The sum $H + rH$ is called the inflationary premium. This value must be added to the real rate of return to compensate for inflationary losses. In russian methodical

recommendations offer a stripped down formula Fischer, namely $r_H = r + H$, that is not entirely justified, since at sufficiently high inflation, when the expense of tens of millions, every percentage point that hundreds of thousands of rubles. In addition it is considered that for example, banks with operations with deposits offer a rate that takes into account the inflation rate (the nominal rate), which is not true.

Example 2.2.

The rate offered by the Bank - 10% annual compound interest with charge 1 time per year. The inflation rate is 12%. What is the real income received by the investor? What should be the interest rate that will actually get the real income of 10 %.

1. Let the interest is calculated once a year. Use the Fisher formula and find out her real rate of return:

$$r = \frac{r_H - H}{1 + H} = \frac{0,1 - 0,12}{1,12} = - 0,01786.$$

We received a negative interest rate, i.e. there is an erosion of capital, i.e. a loss of investor is 1,786%.

2. Let the interest is charged 4 times a year. From the formula (2.16) find the value of the real interest rate :

$$r = m\left(\frac{1 + r_H / m}{\sqrt[m]{J}} - 1\right) = 4 \cdot \left(\frac{1 + 0,1/4}{\sqrt[4]{0,12}} - 1\right) = - 0,01453.$$

Rate is negative, the investor receives a loss 1,453%.

Find the gross rate which provides the desired yield of 10% per annum.

1. Let the interest is calculated once a year. We will use the Fisher formula: =0,1 + 0,12+0,1*0,12=0,232,

that is, the Bank must offer annual rate of loan interest, 23, 2%.

2. Let the interest is charged 4 times a year. Let's use the formula:

$r_H = r + H + rH$ =0,1 + 0,12+0,1*0,12=0,232, .

The Bank might offer a lower rate in 21,78% to get the real rate of return of 10% per annum when calculating compound interest 4 times in the year,.

2.4. INFLATION AND DEFLATION.

Taking into account inflation and deflation are operations, consisting in identifying the real value of economic indicators by increasing or decreasing their nominal value in a changing price level.

The nominal value of the economic indicator is its expression in current prices. The real value is its value, adjusted for inflation (rising prices) or deflation (lower

prices). The real figures are more accurate in comparison with the nominal characteristics of economic processes.

To implement processes of taking into account inflation and deflation are the basic indices of inflation. The inflation index for the current one period is called a *chain*, and the inflation index for the base period is called the *base* and is defined as the product of chain indices for the periods included.

Usually businessmen have an agreement about what period to consider as basic when you compare prices or any cost indicators. To convert nominal values into real, they should be separated on the base the inflation index if the reference period is preceded by the analozed. If the value of the inflation index less than one, the adjustment is the nominal rate increase. If the value of the price index is greater than one, adjusting the nominal rate downward.

If the period is preceded by the base, then to transform indicators to a comparable mind in prices of the base period the researcher should multiply indicators by the basic inflation indices.

Thus, it is possible to give the following definitions:

Taking into account inflation - recalculation of the cost parameters using price of base period.

Taking into account deflation - recalculation of the cost parameters in real prices earlier base period.

Example 2.3. T

There are statistics on GNP of Russia since 1996 and 2011 in current (nominal) prices.

Obtain real GNP Using index-deflator . Choose the end of 2000 for the base point.

Table 2.1. Taking into account inflation and deflation

годы	GDP in current price	J chain	J base	GDP real
1996	2007,83	1,46	3,249	6523,44
1997	2342,51	1,15	2,825	6617,59
1998	2629,62	1,19	2,374	6242,72
1999	4823,23	1,72	1,38	6656,06
2000	**7305,65**	1,38	**1,00**	**7305,65**
2001	8943,58	1,16	1,16	7709,98
2002	10830,54	1,16	1,345	8052,45
2003	13208,23	1,14	1,539	8582,35
2004	17027,19	1,20	1,847	9218,83
2005	21609,77	1,19	2,198	9831,56
2006	26917,20	1,15	2,528	10647,63
2007	33247,51	1,14	2,882	11536,26

2008	41276,85	1,18	3,40	12140,25
2009	38807,22	1,02	3,468	11190,09
2010	46321,78	1,14	3,953	11718,13
2011	55798,67	1,16	4,585	12169,83

For data of GDP in current prices since 2001 use the procedure of deflation (divide the underlying inflation index). For data prior to 2000 use inflaion (multiply the underlying inflation indices).

As can be seen from 2.1, the growth of GNP in current prices much more (as a consequence of rising prices) than the growth of real GNP.
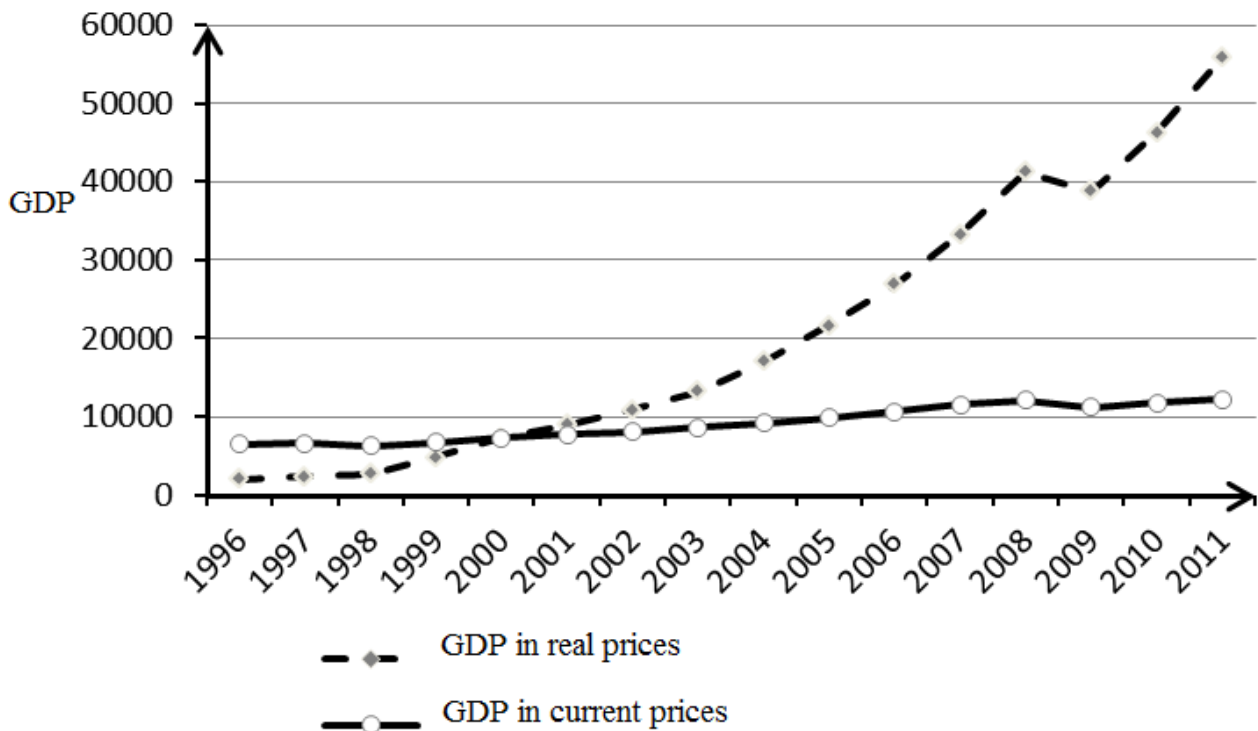
2.1. Dynamics of GDP, billion rubles

4 . THE PAYMENT FLOWS.

4.1. TYPES OF PAYMENT FLOWS AND THEIR PARAMETERS .

The payment flow is a sequence of sums $F_1, F_{2..}, ... F_n$, each of which relate to a specific point in time. The payment flow can be caused by different reasons: the payments of the loan, the fees associated with the investment project, etc. Earlier discussed simplest financial transaction (transaction) can be viewed also as cash flow with two payments: the grant of the loan PV and its repayment FV. Flow elements F_i can be either independent or interconnected in any algorithm.

The payment flows can be regular (the amount of payments constant or may vary in accordance with any algorithm at equal intervals of time) and irregular. Irregular flow members are both positive (inflow) and negative values (payments). Payments can be also made through different time intervals.

Definition. The stream of payments adequate to the following requirements: all members are positive values, and the time intervals between the payments the same; is called a financial rent (rent) or annuity regardless of destination and origin charges.

Rents are often encountered in practice: rent, contributions to repay the loan, pensions and other. Annuity is characterized by a number of parameters, which are divided into basic, additional and generalizing.

Main parameters:

F_i - member of rents (individual payment);

t - the period of rent (the time interval between two consecutive payments);

n - the period of rent (the time from the beginning of the first period rent until the end of the last);

r - interest rate.

Additional options:

m - the number of charges per year;

p - the number of payments per year.

Consider the classification of rent on the basis of the above parameters.

1. The number of payments during the period: annual (1 year) and p-term (p once a year). In financial practice also meet with sequences of payments that are made so often that they can be called continuous.

2. The number of accrual of interest during the year: annual charge, with charge m times of the year, with continuous compounding.

3. Largest members: constant (equal payments) and variables. Members of

variable annuities can modify their sizes over time following any law (arithmetic, geometric progression, etc.) or unprocessed.

4. On the probability of a payout: faithful and conditional.

Faithful rent shall be unconditional payment. The number of members of such rents is known in advance. The payment of conditional rent is dependent on the occurrence of some random event, so the number of members it is not known in advance. These rents include, for example, insurance annuities (various payments in property and personal insurance, lifetime pension payments and other).

5. Number of members: with a finite number of payments, i.e. limited in terms of rent (with a predetermined period), and infinite or eternal rent (perpetuity). Perpetual rents are encountered in practice in a number of long-term operations, when it is assumed that the period of functioning of the analyzed system, or the transaction period is very long and does not specify the specific dates.

6. The ratio of the commencement of rent and any time, proactive beginning of the rent (for example, the beginning of the contract or the date of its conclusion): immediate annuity and deferred (deferred annuity).

7. At the time of disbursement of payments within the period: remunerate (annuity due) payments in the beginning of the period; pastureland (ordinary annuity) payments at the end of the period, uniform - timed payments to the middle of the period.

Generalizing options payment flows.

Analysis and assessment of stream of payments typically involve the calculation of one of two final characteristics: present value *PV* or future value *FV*.

Future value *FV* (amount of cash flows) is the sum of all of members of the stream of payments with accrued to the end of the term interest. In this case, the estimation is based on the positions of the future, the scheme of building capital has implemented.

Present value *PV* (present value of cash flows) is the sum of all of the members of the stream, discounted at the beginning of the rent period or some proactive time. Evaluation of flow is conducted from the perspective of the present, discounting is implemented scheme in this case.

The specific meaning of these characteristics is determined by the content of its members or their origin. Future value may present the total amount of accumulated debt by the end of the period, the total amount of investment accumulated a cash reserve, etc. In turn present value characterizes refer to the beginning of the project investment costs total capitalized income or net present profit, etc.

It should be noted that accumulating and discounting are carried out according to the scheme of compound interest when cash flows is estimated.

4.2. FIND FUTURE VALUE OF PERMANENT RENT

1. Consider a constant annual rent of pastureland with interest accumulated once a year.

In this case, the proceeds made at the end of equal time intervals, which divided the considered period of time. At the same time $F_1 = F_{2..} = ... = F_n = A$. The percentage of income are accumulated compound into interest rate r.
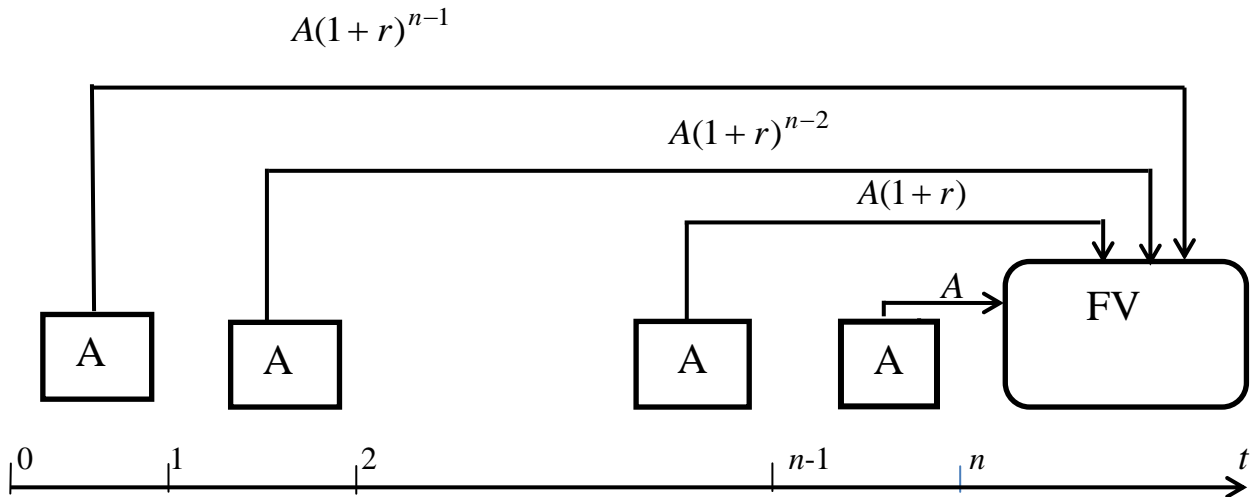
Give a graphic illustration.

Fig. 4. 1. The scheme of formation of the future value of the payment flow.

All members except the last (rent, pastureland) bring interest at the time from receipt of payment until the end of the operation. Then, by definition, accumulated amounts:

$$FV = A + A \cdot (1+r) + A \cdot (1+r)^2 + ... + A \cdot (1+r)^{n-1}. \quad (4.1)$$

The expression (4.1) is the sum of the members of the geometric progression with the first member $a_1 = A$ and the denominator of q=(1+r) equal to the multiplier of the building for one period. Let's use the formula of sum of n members of a geometric progression:

$$S_n = a_1 \cdot \frac{q^n - 1}{q - 1}, \quad (4.2)$$

$$FV = A \cdot \frac{(1+r)^n - 1}{r}. \quad (4.3)$$

The factor multiplied by the member annuity, called accumulation factor and mean s_{nr}. The index n indicates the length of the rent and r - the interest rate, and s_{nr} - can be interpreted as the accrued amount of the annuity with a single payment for

the rent $S_{nr} = \dfrac{(1+r)^n - 1}{r}$. In academic and financial literature sometimes are the tables of the values of this index.

2. The constant annuity of prenumerata

Unlike rent pastureland, the interest is calculated on the last payment:

$$FV_{pr} = A \cdot (1+r) + A \cdot (1+r)^2 + \dots + A \cdot (1+r)^n \quad (4.4)$$

It is obvious that the sum (4.4) in $(1+r)$ times greater than the sum of (4.1)

$$FV_{pr} = A \cdot \dfrac{(1+r)^n - 1}{r} \cdot (1+r)$$

3. The constant annuity with interest m once a year

In this case, the members of the rent accumulate interest adequate to the following series:

$$A, \quad A \cdot \left(1 + \dfrac{r}{m}\right)^m, \quad A \cdot \left(1 + \dfrac{r}{m}\right)^{2m}, \quad \dots A \cdot \left(1 + \dfrac{r}{m}\right)^{(n-1)m}, \quad (4.5)$$

r is the nominal rate of interest.

This series is a geometric progression with denominator $q = \left(1 + {r}/{m}\right)^m$ and the first member $a_1 = A$. Then the sum of terms of a geometric progression by the formula (4.2) will find future value of considered rent:

$$FV = A \dfrac{\left(1 + {r}/{m}\right)^{mn} - 1}{\left(1 + {r}/{m}\right)^m - 1}. \quad (4.6)$$

When considering rent prenumerata interest will accumulate on the last payment. Therefore, the accrued amount for rent prenumerata is at $q = \left(1 + \dfrac{r}{m}\right)^m$ times more than for rent pastureland:

$$FV_{pr} = A \cdot \dfrac{\left(1 + {r}/{m}\right)^{mn} - 1}{\left(1 + {r}/{m}\right)^m - 1} \cdot \left(1 + {r}/{m}\right)^m. \quad (4.7)$$

Following algorithm of formation of the future value of permanent rent, you can obtain a formula for any type of rent (tab. 4.1).

Таблица 4.1

Основные формулы наращенния постоянных рент

Type of rent	Rent postnumerata FV	Rent prenumerata $F\widetilde{V}$

$p = 1, m = 1$	$FV = A \cdot \dfrac{(1+r)^n - 1}{r}$	$F\widetilde{V} = FV \cdot (1+r)$
$p = 1, m \neq 1$	$FV = A \dfrac{\left(1 + r/m\right)^{mn} - 1}{\left(1 + r/m\right)^m - 1}.$	$F\widetilde{V} = FV \cdot \left(1 + \dfrac{r}{m}\right)^m$
$p, m = 1$	$FV = \dfrac{A}{p} \cdot \dfrac{(1+r)^n - 1}{(1+r)^{1/p} - 1}$	$F\widetilde{V} = FV \cdot (1+r)^{1/p}$
$p, m, p=m$	$FV = A \cdot \dfrac{\left(1 + r/m\right)^{mn} - 1}{r}$	$F\widetilde{V} = FV \cdot \left(1 + \dfrac{r}{m}\right)$
$(p, m, p \neq m$	$FV = A \dfrac{\left(1 + r/m\right)^{mn} - 1}{p\left[\left(1 + r/m\right)^{m/p} - 1\right]}$	$F\widetilde{V} = FV \cdot \left(1 + \dfrac{r}{m}\right)^{m/p}$
$p = 1, m \to \infty$	$FV = A \cdot \dfrac{e^{\delta n} - 1}{e^{\delta} - 1}$	$F\widetilde{V} = FV \cdot e^{\delta}$
$p, m \to \infty$	$FV = A \cdot \dfrac{e^{\delta n} - 1}{p \cdot \left(e^{\delta/p} - 1\right)}$	$F\widetilde{V} = FV \cdot e^{\delta/p}$

The difference between regular rent prenumerata from rent pastureland is that time of the accrual of interest on each payment increases by one period of the rent. Therefore, the future value $F\widetilde{V}$ will be more than the sum of FV in $S_{1,r}$ time, where $S_{1,r}$ is the accumulation factor of payment in one period corresponding to this type of annuity (equal to the denominator exponentially for a given stream of payments). Thus, $F\widetilde{V} = FV \cdot q$.

As can be seen from the above formulas, frequency of payments and accumulation of interest can significantly affect at the value of the future value. We denote $FV(p,m)$ as the accrued amount for permanent rent with the number of payments per year p and the number of interest accumulation m. It is possible to show that the following ratio:

$$FV(1,1) < FV(1,m) < FV(1,\infty) < FV(p, 1 | p > 1) < FV(p,m | p > m > 1) <$$
$$< FV(p,m | p = m > 1) < FV(p,m | m > p > 1) < FV(p,\infty).$$

The above inequality can be used when choosing the terms of the contracts, as they allow to get an idea about the priority of certain conditions in advance. To illustrate, let consider a permanent rent pastureland with parameters: n=10, A=10, r= 6%.

	$m=1$	$m=2$	$m=4$	$m=12$	$m=\infty$
$p=1$	131,81	132,37	132,65	132,85	132,95
$p=4$	134,74	135,35	135,67	135,88	135,99

4.3. FINDING THE PRESENT VALUE OF PERMANENT RENT

When it is needed to define present value of a rent, we should assess future cash flows from the position of the current moment and use the mathematical scheme of discounting on the basis of compound interest.

1. Consider the annual rent pastureland with interest once per year ($m = 1$).

Discount each payment at the beginning of the operation and will find the som of these payments:

$$PV = \frac{A}{1+r} + \frac{A}{(1+r)^2} + \ldots + \frac{A}{(1+r)^n} \,. \,(4.8)$$

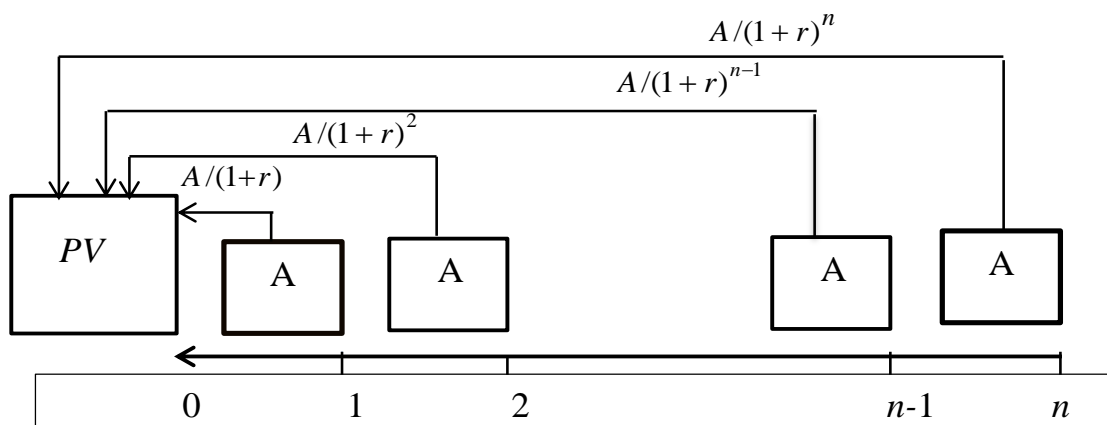Give a graphic illustration (fig. 4.2).

Fig. 4.2. The scheme of formation of the present value of the constant annual rent pastureland.

It is obvious that (4.8) is the sum of *n* members of the geometric progression with the first member $a_1 = \frac{A}{1+r}$ and the denominator $q = \frac{1}{1+r}$. Then by the formula (4.2) present value of the rent will be determined as:

$$PV = A \cdot \frac{1-(1+r)^{-n}}{r} \,. \,(4.9)$$

The factor multiplied by the member's rent is called the discount factor of a rent and is denoted by a_{nr} , where the subscript *n* indicates the period of rent, and $r -$ Interest rate. For the considered rent $a_{nr} = \frac{1-(1+r)^{-n}}{r}$ (the values tabulated).

Example 4.1.

It is proposed to take land on lease for three years with payment of 10 million rubles at the end of each year. Rate of Bank interest - 20% per annum.

Find modern value of the annuity.

$$PV = 10 \cdot \frac{1 - 1{,}2^{-3}}{0{,}2} = 21{,}0648$$

I.e., price of this annuity is 21,0648 million rubles for the current time position. Meaning of this value is simple: this is the initial invested sum which you can deposit at this interest, that will ensure the payment of all rental payments.

2. Perpetual annuity.

The annuity is called perpetual, if cash receipts continue for quite a long time and characterize by unlimited final dates. Only the task of finding the present value makes sense in this case for the stream of payments.

Let's use the formula of the present value of permanent rent, pastureland (4.9).

$$PV = \lim_{n \to \infty} \left(A \cdot \frac{1 - (1+r)^{-n}}{r} \right) = A \cdot \lim_{n \to \infty} \left(\frac{1 - (1+r)^{-n}}{r} \right) = A \cdot \frac{1}{r},$$

$n \to \infty$, so let's move this formula to the limit:

$$PV = \frac{A}{r}. \ (4.10)$$

This formula is used to evaluate the posibility of purchasing an annuity. The iscount factor is usually taken guaranteed interest rate (for example, the rate of the Central Bank).

3. Renta p-term pastureland with the interest accumulation once a year (m=1).

If payments are made not one time per year, but p time a year, the size of the payment amounts $\frac{A}{p}$ and interest is accrued p for periods of time $\frac{l}{p}$, $l = \overline{1, n \cdot p}$, ($n \cdot p$ is the total number of payments). Then we get the following number of discounted payments:

$$\frac{A}{p \cdot (1+r)^{1/p}}, \ \frac{A}{p \cdot (1+r)^{2/p}}, \ldots, \frac{A}{p \cdot (1+r)^{n}} . \ (4.11)$$

This series is a geometric progression with the first member $a_1 = \dfrac{A}{p \cdot (1+r)^{1/p}}$ and the denominator $q = \dfrac{1}{(1+r)^{1/p}}$.

We can get the formula for the modern cost of rent using formula (4.2) of the sum of n members of geometric progression:

$$PV = \frac{A}{p} \cdot \frac{1-(1+r)^{-n}}{\left[(1+r)^{1/p} - 1\right]} \quad . \, . \; (4.12)$$

.

Applying the above technique to find the present value of permanent rent (based on the sum of n members of a geometric progression), it is possible to obtain formulas for all types of permanent rent (Tab. 4.2).

Table 4.2.

Basic formulas to analyze *PV* ofpermanent rent

Type of rent	Rent postnumerata *FV*	Rent prenumerata $F\tilde{V}$
$p = 1, m = 1$	$PV = A \cdot \dfrac{1-(1+r)^{-n}}{r}$	$P\tilde{V} = PV \cdot (1+r)$
$p = 1, m \neq 1$	$PV = A \dfrac{1-\left(1+r/m\right)^{-mn}}{\left(1+r/m\right)^{m} - 1}$	$P\tilde{V} = PV \cdot \left(1 + r/m\right)^{m}$
$p, m = 1$	$PV = \dfrac{A}{p} \cdot \dfrac{1-(1+r)^{-n}}{(1+r)^{1/p} - 1}$	$P\tilde{V} = PV \cdot (1+r)^{1/p}$
$p, m, p = m$	$PV = A \cdot \dfrac{1-\left(1+r/m\right)^{-mn}}{r}$	$P\tilde{V} = FV \cdot \left(1 + r/m\right)$
$(p, m, p \neq m$	$PV = A \dfrac{1-\left(1+r/m\right)^{-mn}}{p \cdot \left[\left(1+r/m\right)^{m/p} - 1\right]}$	$P\tilde{V} = FV \cdot (1 + \dfrac{r}{m})^{m/p}$
$p = 1, m \to \infty$	$PV = A \cdot \dfrac{1-e^{-\delta n}}{e^{\delta} - 1}$	$P\tilde{V} = FV \cdot e^{\delta}$
$p, m \to \infty$	$PV = A \cdot \dfrac{1-e^{-\delta n}}{p\left(e^{\delta/p} - 1\right)}$	$P\tilde{V} = PV \cdot e^{\delta/p}$

The difference between regular rent prenumerata and pastureland is that time of accumulation of interest decreases by one period. Hence, present value of rent prenumerata will be more than the sum of the same rentpastureland in $\dfrac{1}{a_{1r}}$ time, where $a_{1,r}$ is a discounting factor for one period corresponding to this type of annuity (equal to the denominator exponentially for a given stream of payments). Thus,

$$P\widetilde{V} = PV \cdot \frac{1}{q}.$$

4.4. IRREGULAR PAYMENT FLOWS

The unifying characteristics of irregular stream (s) can only be obtained by direct accumulating (discounting) of all members of this stream of payments.

Let $F_1, F_{2..},...F_n$ is the money flow with unequal payments and different periods between their revenues. The date of receipt of the payments t_i is determined by a special graph (Fig. 4.3). Interest is accrued at the rate of compound interest at the end of each period.

$$\begin{array}{cccccc}
t_0 & t_1 & t_2\ldots\ldots & & t_{n-1} & t_n \\
& F_1 & F_2\ldots\ldots & & F_{n-1} & F_n
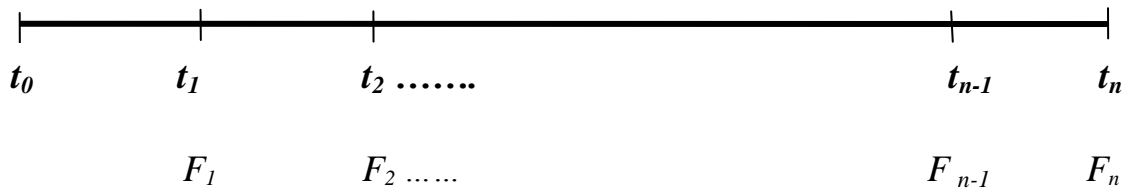\end{array}$$

Fig. 4.3. Schedule of payments

To determine the future value should accumulate interest on every member of this stream for a given interest rate and the selected profile. Members of the stream with the accrued interest form the following series:

$$F_n,\ \ F_{n-1}\cdot(1+r)^{(t_n-t_{n-1})/K},\ \ F_{n-2}\cdot(1+r)^{(t_n-t_{n-2})/K},...F_1\cdot(1+r)^{(t_n-t_1)/K},$$

$(t_n - t_i),\ i = \overline{1, n-1}$ the duration of the interest period for payment F_i (in days) and K the duration in days (calendar).

The sum of this series $FV = \sum\limits_{i=1}^{n} F_i \cdot (1+r)^{\frac{t_n-t_i}{K}}$. (4.13)

To find the present value of the stream will carry out mathematical discounting on the basis of the annual interest rate r. Get discounted stream of payments:

$$\frac{F_1}{(1+r)^{\frac{t_1-t_0}{K}}},\ \frac{F_2}{(1+r)^{\frac{t_2-t_0}{K}}},...,\frac{F_n}{(1+r)^{\frac{t_n-t_0}{K}}},$$

$$PV = \sum_{i=1}^{n} \frac{F_i}{(1+r)^{\frac{t_i - t_0}{K}}} \quad (4.14)$$

If payments are made annually at end of each year, the future value is:

$$FV = \sum_{i=1}^{n} F_i \cdot (1+r)^{n-i},$$

and present value is:

$$PV = \sum_{i=1}^{n} \frac{F_i}{(1+r)^i} \qquad (4.15)$$

Example 4.2.

Find (PV and FV) for the following payment flow (Fig. 4.4), if: the interest rate -10% APR and payment schedule..
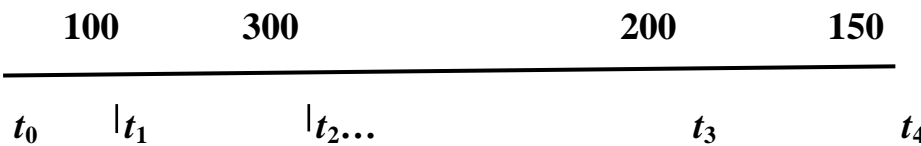
100	**300**		**200**	**150**
t_0	t_1	t_2...	t_3	t_4

Fig. 4.4. Diagram of payments flow.

Table 4.3. Payment schedule

	t_0	t_1	t_2	t_3	t_4
даты	01.01.2013	01.03.2013	30.06.2013	15.09.2013	01.12.2013
t_n-t_i (дн.)	0	275	154	75	0
t_i-t_0 (дн.)		59	180	257	334

$$FV = 150 + 200 \cdot (1+0{,}1)^{\frac{75}{365}} + 300 \cdot (1+0{,}1)^{\frac{154}{365}} + 100 \cdot (1+0{,}1)^{\frac{275}{365}} = 773{,}82.$$

$$PV = \frac{100}{(1+0{,}1)^{\frac{59}{365}}} + \frac{300}{(1+0{,}1)^{\frac{180}{365}}} + \frac{200}{(1+0{,}1)^{\frac{257}{365}}} + \frac{150}{(1+0{,}1)^{\frac{334}{365}}} = 709{,}19.$$

## 4.5. TWO-WAY FLOW OF PAYMENTS. THE NET PRESENT VALUE.

Financial transactions may involve multiple and multi-temporal transitions money from one owner to another. Considering the stream of payments from the position of one of them, you can read everything submitted to him as a positive value, and all payments as a negative value. To assess the financial operations generally used indicator of the net present value (NPV), calculated by the formulas (4.14) or

(4.15), but given the character of each member of the stream.

Example 4.3.

The contract between the company and the Bank stipulates that the Bank provides for 2 years loan for the company in annual installments 2 and 1 million at the beginning of each year at the rate of 10% per annum. The firm repays the debt by paying 2, 1, 2 millions consistently at the end of 3, 4 and 5 years.

What NPV is a Bank?
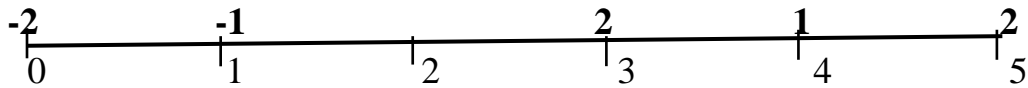
$$NPV = -2 - 1 \cdot \frac{1}{1,1} + 2 \cdot \frac{1}{(1,1)^3} + 1 \cdot \frac{1}{(1,1)^4} + 2 \cdot \frac{1}{(1,1)^5} = 0,518 \text{ млн.дол.}$$

NPV can be interpreted as the investor's profit. Since the result is positive, this operation is acceptable for the Bank.

The requirement of positivity of NPV is required when we make decisions about implementation of the financial transaction by the lender. However, the calculation cannot determine the rationality of such operations, which should be considered when there are alternatives.

The concept of internal rate of return *IRR* (internal efficiency, profitability) is used in such purposes, as the value of the interest rate, which make NPV equal to 0. The effective rate (IRR) is as the root of the equation:

$$\sum_{i=0}^{n} F_i \frac{1}{(1+r)^i} = 0 \text{ - the equation of profitability.}$$

This definition of effective rate of any financial transaction generalizes the concept of effective rate of the elementary operations given previously.

Indeed, in the simplest operations: $F_1 = -PV$, $F_2 = FV$, period of operation *T*. In this case, the equation yields can be written:

$$-PV + FV \cdot \frac{1}{(1+r)^T} = 0 \rightarrow (1+r)^T = \frac{FV}{PV} \rightarrow r = IRR = \left(\frac{FV}{PV} \right)^{\frac{1}{T}} - 1,$$

It is correspond with the definition of effective rate given in section 1.2.

Thus, for any financial transactions with clearly terms and sums of mutual payments can be measured effectiveness, as a percentage provides NPV (the net present value of the stream of payments) equal to zero. Choosing between different possible financial transactions, the investor is always focused on the action with the highest effective rate *IRR*.

Theorem 1. If all negative payments precede all positive, or Vice versa, then the equation of profitability has a positive root r_0 is the internal rate of return IRR.

Theorem 2 (generalizes the previous one).

Let $t_0 < t_1 < \ldots < t_n$ и $C_m = \sum_{s=1}^{m} C(t_s)$, $m = \overline{1,n}$ - the cumulative sum of all net payments to the investor from time 0 until time t_m inclusive.

If $C_0 \neq 0$, $C_n \neq 0$ form a flow of payments $(C_0, C_1, \ldots, C_n)$ and this flow has exactly one change of sign(after exclusion of zero values), the equation of profitability has only one positive root.

Example 4.4.

May have the following stream of payments.

Find the IRR.

t	0	1	2	3	4
F_t	-5	1	-3	8	4

Make the table of accumulated flow net payments:

t	0	1	2	3	4
C	-5	-4	-7	1	5

As the accumulated flow net payments has only one change of sign, then we can form equation of profitability and it has 1 root (by theorem 2):

$$NPV(r) = -5 + \frac{1}{1+r} - \frac{3}{(1+r)^2} + \frac{8}{(1+r)^3} + \frac{4}{(1+r)^4} = 0.$$

The single positive root $r_0 = 22,1\%$.

4.6. DETERMINATION THE PARAMETERS OF THE PERMANENT RENT.

As it was shown above, permanent rent is described by a set of key parameters: basic A, r, n and additional parameters p, m. However, the designing of contracts and financial transactions can occurs situations, when you set one of the two final characteristics *PV* or *FV* and two basic parameters. There is a need to calculate a value for the missing parameter.

To solve this problem, choose a formula *PV* or *FV* for a specified rent (depending specified characteristics). The result is an equation with one unknown parameter, which should be found as unknown element of this equation.

Time period of annuities calculation.

Sometimes there is a need to determine the term of rent and the number of members of the rent (accordingly) when developing a contract. It is needed to consider the formula of *PV* and *FV* rent.

Consider the annual rent (m=1, p=1):

$$FV = A \cdot \frac{(1+r)^n - 1}{r}, \rightarrow (1+r)^n = \frac{FV \cdot r}{A} + 1 \rightarrow n \cdot \ln(1+r) = \ln(\frac{FV \cdot r}{A} + 1) \ ,$$

$$n = \frac{\ln(\frac{FV \cdot r}{A} + 1)}{\ln(1+r)} . (4.18)$$

Similarly, there are formulas for calculating the term for other types of rent at the given size of present value (or future value).

Researchers must take into account the following points when calculating the period of rent:

1. The estimated value of the term will be fractional typically. It is necessary to round the result. In these cases, it is often more convenient to take *n* as the closest smallest figure for annual rent. For a p-term of rents, the results are rounded to the nearest whole number of periods *n*p* .

For example, for quarterly rent we received n=6,28 year, then n*p=25,62 quarter. Rounded to 25, in this case, the period n=6.25 years.

2. If the result is rounded to the lower integer number, the future value of the rent with this term become less than specified before (in the contract). There is need to calculate an appropriate compensation. For example, if we are talking about the repayment of the debt by payment of permanent rent, compensation may be made by the appropriate payment at the beginning or end of the term, or by increasing the amount of the rent member.

Discuss another problem associated with the term rent. Let PV be the current value of the debt. If the debt is repaid by permanent rent, then

$$n == \frac{\ln(1 - \frac{PV}{A} \cdot r)^{-1}}{\ln(1+r)} .$$

It follows that the debt can be repaid over a finite number of years only, provided that $A > PV \cdot r$. If, however, $A < PV \cdot r$, then the debt has been increasing systematically.

Example 4.6.

What is the time required to accumulate 100 million rubles, provided that 1 million rubles is played each month and the accumulation with interest at the rate of 25% per annum?

We have $\dfrac{A}{p} = 1$, $\quad p = 12$, then $A = 12$. Use the formula for the sum for p-term rent:

$$FV = A \cdot \frac{(1+r)^n - 1}{p \cdot [(1+r)^{1/p} - 1]}, \ (4.17)$$

$$n = \frac{\ln\left\{\dfrac{FV}{A} \cdot p[(1+r)^{1/p} - 1] + 1\right\}}{\ln(1+r)} = \frac{\ln\left\{\dfrac{100}{12} \cdot 12[(1+0.25)^{1/12} - 1] + 1\right\}}{\ln(1+0.25)} = 4{,}7356 \ \text{г.}$$

If the period is rounded up to five years, it is necessary to reduce the size of the annuity member, i.e. to find a member of the rent for n =5.

We find from the formula (4.17) the amount of monthly payment $\dfrac{A}{p}$ for n=5:

$$\frac{A}{p} = \frac{S \cdot ((1+r)^{1/p} - 1)}{(1+r)^n - 1} = \frac{100 \cdot (1{,}25^{1/12} - 1)}{1{,}25^5 - 1} = \frac{1{,}8769}{2{,}0517578} = 0{,}914776 = 914{,}776 \ \text{тыс.руб.}$$

Consequently, the monthly payment must be 914,776 thousand rubles.

Member of rent calculation.

Initial conditions: set *FV* or *PV* and the set of parameters excluding *A*. For example, for a specified number of years *n* is required to establish a Fund in the sum of S by systematic regular contributions. If it is assumed that the rents should be annual, pastureland, with annual interest, then, applying the formula of the future value, rent will receive:

$$A = \frac{S}{s_{n,r}}. \ (4.16)$$

It is easy to find member of rent *A* for rent with any conditions at the base of formulas for future value *FV* .

Let present value is known now (specified by the terms of the contract). If the annual rent, pastureland, m= 1, then from the formula present value (4.9) follows:

$$A = \frac{PV}{a_{n,r}}. \ (4.17)$$

It is easy to identify member of rent *A* and for other conditions of rent formula-based calculations.

Example 4.7.

Define the member of rent for the periodic contributions when the solution of the following two tasks are made:

a) it is established a special Fund (e.g., for debt repayment or investment) in the amount of 100 million rubles;

b)it is needed to repay current debt in installments in the sum of 100 million rubles.

The term in both cases is five years, the interest rate is 20%, annual payments of pastureland.

a) $FV = 100 = A \cdot \dfrac{(1+0,2)^5 - 1}{0,2}, \rightarrow A = \dfrac{100}{7.4416} = 13,438$

б) $PV = 100 = A \cdot \dfrac{1 - (1+0,2)^{-5}}{0,2} = A \cdot 2.9906, \rightarrow A = \dfrac{100}{2,9906} = 33,438$

Calculation of interest rate.

The need to determine the interest occurs whenever it comes to determe the effectiveness (profitability) of the financial-banking operations. In the simplest case, the task is formulated as follows: to solve an equation $FV = A \cdot \dfrac{(1+r)^n - 1}{r}$ or

$PV = A \cdot \dfrac{1 - (1+r)^{-n}}{r}$ relatively to A. It is easy to see that the algebraic decision no.

To obtain the desired index use linear interpolation or any iterative method such as the method of Newton - Raphson, secant method, etc. to avoid the use of a computer with the appropriate software package (for example,if it is impossible).

Linear interpolation:

A, FV or *PV* are set. Our subject is to find the values of the accumulation or discount factor:
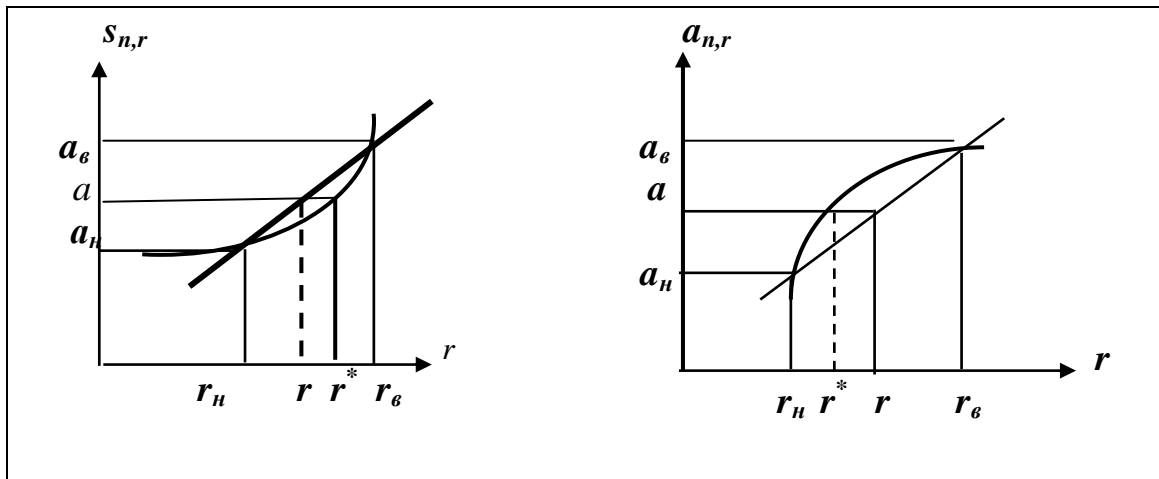
$$s_{n,r} = \frac{FV}{A} \qquad a_{n,r} = \frac{PV}{A}$$

Fig. 4.5. Graphical illustration of the method of linear interpolation

We should use the following interpolation formula to estimate r:

$$r = r_{_н} + \frac{a - a_{_н}}{a_{_в} - a_{_н}}(r_{_в} - r_{_н}), \quad (4.19)$$

$a_{_в}$ and $a_{_н}$ - values of the accumulation or discounting factor for the upper and lower values of the interest rate (the interest rate $r_{_в}$, $r_{_н}$), and a is the the accumulation or discounting factor for which is determined the size of the bet.

The following illustrations depict the dependencies of the corresponding coefficients on the interest rate level, the interpolation estimates and their exact values. The first is designated as r , the second - as r^*.

As can be seen (fig. 4.5), the interest rates differ slightly from the exact values of this quantity, and if based on the discounting factor, the estimate is overstated, in turn, the evaluation of r by the accumulation factor is less accurate values. The smaller the range ($r_{_в}$, $r_{_н}$) the more exact assessment of the interest rate.

Example 4.8.

It is assumed that annual contributions of pastureland 100 million rubles for seven years to create a Fund in the amount of 1 billion rubles.

What should be the annual interest rate?

Determine the initial ratio of building: $s_{7,r} = 1000 / 100 = 10$.

Suppose that the required interest rate is in the range of 11-12%. For these values of the bet find the accumulation factor: $a_{_в} = s_{7,12} = 10{,}08901$, $a_{_н} = s_{7,11} = 9{,}78327$.

Here $r = 0{,}11 + \dfrac{10 - 9{,}78327}{10{,}08901 - 9{,}78327} \cdot (0{,}12 - 0{,}11) = 0{,}11709$ or $11{,}709$ %.

Checking: find the accumulation factor:

$$s_{nr} = \frac{(1+r)^n - 1}{r} = \frac{1{,}11709^7}{0{,}11709} = 9{,}999$$

Thus, the found value of the rate ensures the fulfillment of conditions almost exactly.

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